# Modelling Double-Moderated-Mediation & Confounder Effects Using Bayesian Statistics

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**Abstract**
This study provides an example on how to conceptualize and estimate models when double moderated mediation with nominal and continuous (Likert type) variables need to be simultaneously accounted for, and also how to appease reservations given the biases due to the implicit sequential ignorability assumption (endogeneity) regularly overseen in marketing research. We explain the issues and apply the proposed solution using empirical data.
The benefits for research are considerable as this approach is superior to other approaches (e.g. splitting the sample by the binary moderator and estimating a moderated mediation model) while also accounting for accounted confounders.

## Introduction

Researchers may regularly face 2 important empirical estimation problems that have substantial *modelling*, *estimation* but also *theory-development* implications.

Our first empirical problem relates to the estimation of indirect (mediation) effects in the context of more than one moderation influences when one is nominal and the second is continuous. Researchers have been traditionally interested on whether an antecedent/ treatment variable (X) influences an outcome (Y) via a post-treatment/intervening mediator variable (M) and how such influence varies between groups (for example, men and women) (see Muller *et al.,* 2005 for a review of the topic). Substantial work has been conducted by Preacher, Hayes and colleagues (see for instance http://www.quantpsy.org/pubs.htm or http://www.afhayes.com/ for a list of their works; also see Preacher *et al.*, 2007; Preacher & nd Hayes, 2008). However, the possibility to *simultaneously* test for a continuous second moderator was not easy and theory development is curtailed if singular parameter estimates are only produced (i.e., without considering the effects of other *simultaneously operating*, elements). The existence of 2 such simultaneous conditional (moderation) influences within the context of mediation models is commonly faced by researchers and we demonstrate how to model and test these. These can also be the case when researchers have 2 moderators of different level (macro and micro).

Our second empirical problem relates to the substantive assumptions implicitly made in the past to identify direct and indirect effect when mediators are modelled; this is a problem that plagues extant research in management (Antonakis *et al.*, 2010). The *validity* of commonly used mediation analysis based on structural equation models (SEM) critically relies upon this so-called sequential ignorability *assumption* which produces *biased* results (Imai *et al.*, 2010a; 2010b). Such reservations must be appeased to secure robustness of results and we demonstrate how to, in our context case, *adjust* our mediation estimates. We demonstrate the *extent of bias* that would have been the case, and distortion of resultant theoretical advances, if such assumption wouldn't have empirically been accounted for.

The reasons for which sequential ignorability aspects confound the mediation estimates and distort the resulting theoretical advances are explained below. Participants' attribution of scores to questions on outcomes and predictors mean that counterfactual outcomes can never be observed (see Imai *et al.*, 2011) and they are inherently an unobservable quantity. Next, antecedent and mediator variables selected by researchers are not usually randomly selected and the possibility of other covariates which confound the results should not be precluded either (see Imai *et al.*, 2011). Next, even if the antecedent and mediator variables were randomized, the mediation effects cannot be identified unless an additional assumption, namely a no-interaction effect between antecedent and mediator constraint is imposed (Robins, 2003; Muthen, 2011). Last but not least, even when these have been accounted for, the degree of confidence in the results is not known. A sensitivity test is needed to identify higher and lower bounds so to provide the degree of numerical sensitivity regarding the estimates. Without testing for unobserved covariates research in uninformed regarding the distortion in the results and theory development is biased. We demonstrate how to test for these effects and calculate the sensitivity of the estimates.

In doing so, our data come from a dataset regarding professionals (individuals)' willingness to implement a new policy (set of actions). Our case example uses 5 variables acting as follows:

- As **dependent/outcome** (*Y*) the willingness of individuals to implement the activity
- As **independent** (*X*) others' opinion (this is social norms)
- As the **2 moderators** context (1st moderator Group *A*/Group *B*) & 2nd moderator (*MOD*) own satisfaction.
- As a **mediation variable** (*M*) societal meaningfulness.

Our context is just *an example* and should not be treated as singular. Consider in marketing an alternative case of sales and marketing individuals and the imposition of new company practice. A context in consumer behaviour can be individual consumers who will be the subject to common social norms whose reaction is mediated by a variable *M* but at the same time belong to two different age groups and subject to divergent promotion incentives. Investigation of marketing theories can uncover a number of similar contexts.

**The double moderated mediation modelling issue**

The traditional approach commonly employed by researchers in the case of mediational influences with the presence of a *single* moderation is clearly not applicable when the theoretical stance requires the simultaneous estimation of *two* independent conditional processes each affecting the mediation influence in its own right. The problem is exacerbated if one of the conditional processes is a nominal and the other is measured through a continuous (Likert type) variable. A solution to the above model specification and how it can be applied is delineated next. We suggest that the conceptual model is specified as a double moderated mediation model which can be summarized by two regression equations, the first regression equation predicting the outcome Y using our modeled 4 predictors as follows:

$$Y = \beta_0^g + \beta_1^g \cdot M + \beta_2^g \cdot X + \beta_3^g \cdot MOD + \beta_4^g \cdot XxMOD + e_1^g \quad (1)$$

Here, *X* refers to the independent variable; *M* refers to our Mediator, *MOD* refers to the 2nd of our moderating contexts, and g refers to the multiple group structure (our 1st moderator), which in our case context g={1,2} results in estimates for Group *A* and Group *B* separately. For example, $\beta_1^1$ refers to the regression coefficient between *M* and *Y* for the group A, whereas $\beta_1^2$ refers to the same association for Group B. The residual error variances per group, denoted by $e_1^g$ are assumed to be normally distributed with a mean of zero. The second regression equation predicting the mediator $M$ is:

$$M = \gamma_0^g + \gamma_1^g \cdot X + \gamma_2^g \cdot MOD + \gamma_3^g \cdot XxMOD + e_2^g \qquad (2)$$

which can be rewritten as

$$M = \gamma_0^g + (\gamma_1^g + \gamma_3^g \cdot MOD) \cdot X + \gamma_2^g \cdot MOD + e_2^g \qquad (3)$$

The moderation (*MOD*) on the direct effect of $X$ on Y can then be written as

$$\beta_2^g \cdot \beta_4^g MOD \qquad (4)$$

and the indirect effect via Societal Meaninglessness can be written as

$$\beta_1^g \cdot (\gamma_1^g + \gamma_3^g \cdot MOD) \qquad (5)$$

Because the moderated mediation, shown in Equation (5), also contains the group indicator $^g$, these allow the estimation of a double moderated mediation effect when one of the moderators is a nominal group membership variable (profession) whereas the second moderator is a continuous measured variable. To note however that Y and M can be latent variables instead of manifest variables as expressed in Equations (1)-(5).

**The sequential ignorability assumption modelling issue and sensitivity of effects**

We will now turn to the second important empirical problem which has substantial implications for the theoretical outcomes of the present endeavor. The classical mediation analysis (usually based upon Baron & Kenny, 1986; and MacKinnon *et al.,* 2002; 2007) as carried out in structural equation modeling (e.g., Bollen, 1989) **is seriously challenged**. The produced direct and indirect effects through the traditional method may not *actually* be inferred as causal (Holland, 1988, Sobel, 2008). The issues at stake are important. Valeri and VanderWeele (2011) explain the assumptions as: (i) no unmeasured confounding of the treatment (antecedent)-outcome relationship exists; (ii) no unmeasured confounding of the mediator-outcome relationship exists; (iii) no unmeasured treatment-mediator confounding exists; (iv) no mediator-outcome confounder affected by treatment exists. Prudence also implies that even though the influence of additional processes has been already simultaneously accounted, no defense exists that *there is no* other (*non accounted for*), covariate confounders. Assumption (iv) is likely to be violated even in random observational data. The key concept rooted in the causal effects literature, namely the counterfactual (also see Pearl, 2001; 2009; 2012) implies that the outcome $Y_i$ given a score observed for the antecedent variable X (thus $Y_i$ ($x$)) may not be the outcome observed, and is therefore possibly counterfactual (Holland, 1998; Sobel, 2008; Bullock *et al.*, 2010). Researchers specifying their mediation models are strongly advised to assume as a regular course of action that their causally defined direct and indirect mediational effects do indeed violate the ignorability assumption and results should be assumed biased thus distorting theory development. This is an issue that plagues research as also identified by Antonakis *et al.* (2010). Causally defined effects can only be achieved by conducting additional analyses and subjecting the specified models to further constraints (see also Muthen, 2011: 3). Emsley *et al.* (2010), Imai *et at.* (2010a; 2010b; 2011) and Muthen (2011) propose different methods to account for the potential confounding effects of unobserved covariates in the mediation effects, but most, including VanderWeele (2010), agree that this is not enough and the effort has to be concluded with even further additional sensitivity analyses to study for the *extent* of impact from violation of the assumptions. We implement the Muthen (2011) procedure to measure the impact of unobserved covariates and a sensitivity analysis for its lower and upper boundary.

**Analysis and Results**

The total population for our sampling frame consisted of 5,199 individuals, all members of the two main Dutch professional associations (Group A and Group B). Using Mplus v7 (Muthén & Asparouhov, 2012; Muthén & Muthén, 1998-2010), we analyze the perceptions and attitudes of Group A and Group B using Bayesian *credibility* intervals (*CI*) (Gelman *et al.*, 2004) (also see Yuan & MacKinnon, 2009) instead of the maximum likelihood based *confidence* intervals. We constructed a series of models (see table 1).

**Table 1: Structural Paths: Unstandardized (standardized) parameter estimates per group (A= Group A; B= Group B)**

| | | ***g*=1 Group A** | | ***g*=2 Group B** | |
|---|---|---|---|---|---|
| **Regression Equation notation** | **M*plus* notation** | **β (b)** | **b 95% C.I.** | **β (b)** | **b 95% C.I.** |
| | | **Model 0 (No Mediation)** | | | |
| $\beta_0$ | Intercepts *Y* | .15* ( .21*) | .09-.33 | 0[a] | - |
| $\beta_2$ | *Y* ON *X*; | 1.61* (.42*) | .35- .48 | 1.45* (.37*) | .29- .45 |
| $e_1$ | Residual Variances *Y* | .42* (.82*) | .76- .87 | .42* (.86*) | .79- .91 |
| *Explained $R^2$ of Y* | | .17 | .12-.23 | .14 | .08-.20 |
| | | **Model 1 (Simple Mediation)** | | | |
| $\beta_0$ | Intercepts *Y* | .05 (.07) | -.04- .18 | 0[a] | - |
| $\beta_1$ | *Y* ON *M*; | -.35* (-.47*) | -.54- -.40 | -.46* (-.56*) | -.61- -.49 |
| $\beta_2$ | *Y* ON *X*; | 1.03* (.27*) | .19- .34 | 1.24* (.30*) | .22- .37 |
| $\gamma_0$ | Intercepts *M* | -.30* (-.32*) | -.44- -.20 | 0[a] | - |
| $\gamma_1$ | *M* ON *X* | -1.76* (-.34*) | -.41- -.27 | -.60* (-.12*) | -.20- -.03 |
| $e_1$ | Residual Variances *Y* | .30* (.60*) | .54- .67 | .30* (.55*) | .49- .62 |
| $e_2$ | Residual Variances *M* | .78* (.87*) | .82- .92 | .78* (.98*) | .95- .99 |
| *Explained $R^2$ of Y* | | .39 | .32-.45 | .45 | .37-.51 |
| *Explained $R^2$ of M* | | .12 | .07-.17 | .01 | .001-0.04 |
| Indirect (mediation effect) (X->M->Y) | | .62* | | .28* | |
| | | **Model 2 (Double Moderated Mediation)** | | | |
| $\beta_0$ | Intercepts *Y* | .03 (.05) | -.05- 1.16 | 0[a] | - |
| $\beta_1$ | *Y* ON *M*; | -.35* (-.47*) | -.54- -.40 | -45* (-.55*) | -.61- -.48 |
| $\beta_2$ | *Y* ON *X*; | .97* (.25*) | .17- .32 | 1.17* (.28*) | .21- .35 |
| $\beta_3$ | *Y* ON *MOD*; | .03 (.04) | -.03- .12 | .08* (.09*) | .02- .16 |
| $\beta_4$ | *Y* ON *X*x*MOD* | .22 (.05) | -.02- .13 | .24 (.05) | -.01- .12 |
| $\gamma_0$ | Intercepts *M* | -28 (-.30) | -.42- -.18 | 0[a] | - |
| $\gamma_1$ | *M* ON *X* | -1.68* (-.32*) | -.39- -.24 | -.53* (-.10*) | -.19- -.01 |
| $\gamma_2$ | *M* ON *MOD* | -.12* (-.11*) | -.19- -.02 | -.11* (-.10*) | -.19- -.02 |
| $\gamma_3$ | *M* ON *X*x*MOD* | .17 (.03) | -.05- .11 | -.31 (-.05) | -.14- .02 |
| $e_1$ | Residual Variances *Y* | .30* (.62*) | .55- .68 | .30* (.55*) | .48- .62 |
| $e_2$ | Residual Variances *M* | .78* (.87*) | .82- 92 | .78* (.97*) | .94- .99 |
| *Explained $R^2$ of Y* | | .37 | .31-.44 | .44 | .38-.51 |
| *Explained $R^2$ of M* | | .12 | .08-.17 | .02 | .008-.06 |
| | | **Model 3 Sensitivity of Mediation Effects** | | | |
| mediation pathway: *g1X*β1* | | .59* | .42-.78 | .24* | .04- .45 |
| Independent pathway: *g1xz*β1* | | -.06 | -.23- .10 | .13 | -.06- .35 |
| Independent pathway: *g1MOD*β1* | | .04* | .01- .08 | .05* | .01- .09 |
| *Explained $R^2$ of Y* | | .20 | .14-.26 | .16 | .10-.22 |
| *Explained $R^2$ of M* | | .13 | .08-.19 | .03 | .00-.06 |

| Fit indices | Model 0 | Model 1 | Model 2 | Model 3 |
|---|---|---|---|---|
| *Df* | 15 | 35 | 50 | 45 |
| Bayesian Posterior Predictive *p*-value | 0.001 | 0.000 | 0.000 | 0.000 |
| Deviance (DIC) | 9703.891 | 17264.512 | 18371.851 | 18.003 |
| Estimated number of parameters (pD) | 11.951 | 25.335 | 50.764 | 32.499 |
| Bayesian (BIC) | 9784.543 | 17457.440 | 18623.651 | 18251.856 |
| Group A | | | | |
| Posterior Predictive *p*-Value | 0.055 | 0.000 | 0.000 | |
| Deviance (DIC) | 4013.032 | 8430.328 | 10403.997 | |
| Estimated number of parameters (pD) | 8.975 | 3.529 | 18.579 | |
| Group B | | | | |
| Posterior Predictive *p*-Value | 0.011 | 0.000 | 0.000 | |
| Deviance (DIC) | 3825.896 | 7051.773 | 7885.651 | |
| Estimated number of parameters (pD) | 6.111 | 28.655 | -3.559 | |

[a] these parameters are fixed to zero so that they can serve as a reference category.

Model 0 (direct effects only) identified that when the independent variable are in favor of the actions (high *X*), this was positively associated to *Y* both for Group A (*b*=.42; 95% *CI*=.35-.48) and Group B (*b*=.37; 95% *CI*=.29–.45) (see Table 1). Model 1 measures both the direct effects and indirect effects through the mediator (*M*) and estimates the *mediating* effect of *M,* but importantly does so for each group individually as well as *simultaneously* for our 2nd moderator (*MOD*). In Model 3 we tested for the endogeneity effect while simultaneously keeping all mediation and moderation influences present. We employed a procedure developed by Muthen (2011) to test simultaneously for the confounding impact of ignored covariates as well as the sensitivity of the estimates and we did this for all three distinct pathways of covariates' impact, namely from *X* (coded *g1X*β1*), the 2nd moderator (*MOD*) (coded *g1MOD* β1*) and their interaction (*X*MOD*) (coded g1xz*β1) upon *Y* through the mediator *M*. The logic behind the distinct treatment of each pathway was that ignored covariates affect each pathway separately, so these need to be tested simultaneously, but each one on individual basis. The reduction in the mediation effect due to ignored covariates is apparently not changing drastically the theoretical results when no moderation effects are considered. Nonetheless, the explained variances **are substantially reduced** for both Group A ($R^2$=20% (from 37%); 95% *CI*=14.5–26.8%) and Group B ($R^2$=16% (from 44%); 95% *CI*=10.2-22.2%) suggesting that ignoring endogeneity leads to substantial biases and in our case additional profession-related variables play a strong role in the second group. Clear effects of Group A/B context moderation exist regarding the mediation pathway (their 95% *CI* do not overlap although they are adjacent at the value of .42-.45). The above decrease in variance is 28% compared to a smaller decrease of 17% for group A.

## Conclusions

The benefits for research are considerable as this approach is superior to other approaches (e.g. splitting the sample by the binary moderator and estimating a moderated mediation model) while also accounting for accounted confounders. In doing so, all estimations are conducted simultaneously while also adjusted for the effects of non accounted confounders for, an issue that currently plagues extant research.

Note: The code used for the estimation is available from the first author upon request.

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